Haldane Gap

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The S=1 quantum antiferromagnetic chain with the Hamiltonian

$H = \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \lambda\, S_i^z S_{i+1}^z + D\,(S_i^z)^2$ is studied.

Haldane argued that a quantum Heisenberg antiferromagnetic chain with an integer spin has a unique disordered ground state and a finite excitation gap, while the same model with a half-odd-integer spin has a critically ordered unique ground state and no excitation gap. The conjecture was rather surprising (at least) for the following two reasons.

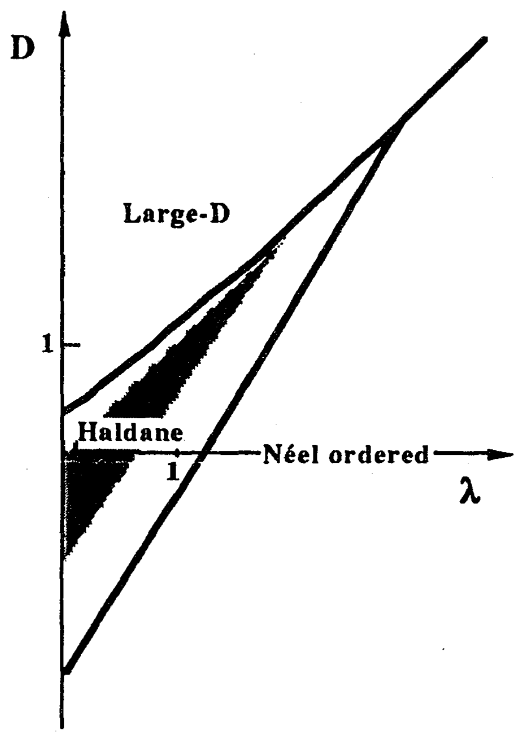

Figure. Numerically obtained phase diagram for the ground state of the S=1 Heisenberg antiferromagnet with uniaxial anisotropy. In the shaded region, we have a proof that the Haldane phase exists in the approximate model with a restricted Hilbert space. We show that the Large-D phase, the Néel phase, and the Haldane phase can be interpreted as Gas, Solid and Liquid, respectively.

i) Such a qualitative dependence on S of the ground state property was against the "universality" principle widely believed among statistical physicists.
ii) When a rotationally invariant quantum system has a unique ground state, one usually expects that there are excitation modes with arbitrarily long wave length and thus with arbitrarily small excitation energy. Haldane's conclusion for the integer S chains was completely against this conventional wisdom.

Haldane's original conjecture was based on approximate field theory mappings in the large-S limit. It was not clear whether the conclusion applies to small values of S, such as S=1. The problem has fascinated theoretical and experimental physicists, and has been studied by various methods. Numerical calculations and experimental results in quasi one dimensional systems seem to be consistent with the conjecture. There are also rigorous results which support Haldane's picture. *But the problem for the finite S Heisenberg model still remains to be understood theoretically.*

The purpose of the present talk is to develop a new picture of the Haldane gap which directly deals with the $S=1$ model. We don't use any field theories, but make use of a (path integral type) stochastic geometric representation of the system. We find that the exotic features of the Haldane phase can be characterized in terms of the language of percolation.

We argue that the Haldane phase with a unique disordered ground state and a gap can be regarded as "liquid". The large-D phase and the Néel ordered phase are identified as "gas" and "solid", respectively. We also introduce a new order parameter that distinguishes the Haldane phase from other disordered phases.

References

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